To rescue a star

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Abstract

Massless neutrinos are exchanged in a neutron star, leading to long range interactions. Many body forces of this type follow and we resum them. Their net contribution to the total energy is negligible as compared to the star mass. The stability of the star is not in danger, contrary to recent assertions.

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It has been recently claimed that the multibody exchange of massless neutrinos renders neutron stars unstable, as the induced self-energy exceeds the mass of the star [1]. Multibody neutron potentials are generated. The putative culprits would be the long distance, infrared, effects associated to neutrino exchange among 4 neutrons or more. Specifically, the more neutrons are involved in a given potential, the larger contribution to the energy would result. This would be an exceptional case in physics where the main contribution would not be dominated by the combinatorial effect of few body potentials.

Long range forces mean infrared effects. The infrared scale of the problem is the radius R of the star. Additional parameters of the problem which may play a significant role are the strength of the weak interactions, G_F, the neutron density, n_n, and a possible neutrino density, n_ν:

$$G_F = 1.17\,10^{-5}\,\mathrm{GeV}^{-2}, \qquad n_n \sim 0.4\,\mathrm{fm}^{-3}$$

$$R \sim 10\,\mathrm{km}, \qquad n_\nu \sim 4.\,10^{-23}\,\mathrm{fm}^{-3} \tag{1}$$

We first analyze the problem in the Hartree-Fock approximation: correlations among neutrons are neglected. That is, it will be assumed that the neutrons are uniformly distributed in the star.

1 Neutrino spectrum in a neutron star

The properties of neutrinos and antineutrinos that propagate in a medium differ from those in the vacuum. In particular, the vacuum energy-momentum relation for massless fermions, $\omega = |\vec{q}|$, where ω is the energy and $|\vec{q}|$ the magnitude of the momentum vector, does not hold in a medium [2].

Assume for the time being that the material of the neutron star is made exclusively of neutrons, among which neutrinos are exchanged. The density-dependent corrections to the neutrino self-energy result, at leading order, from the evaluation of Z^0-exchange diagrams between the neutrino and the neutrons in the medium, with the Z^0 propagator evaluated at zero momentum. They can be summarized [3] by the following dispersion relation

$$\omega = |\vec{q}| \pm b, \tag{2}$$

where

$$b \simeq -\sqrt{2} G_F\, n_n/2 \sim -0.2\,10^{-7}\,\mathrm{GeV} \sim -10^{-7}\,\mathrm{fm}^{-1} = -10^{-2}.\,\mathrm{\AA}^{-1} \tag{3}$$

In eq. (2) the upper(lower) sign refers to neutrinos (antineutrinos). b resums the zero-momentum transfer interaction of a massless neutrino with any number

of neutrons present in the media. Sensibly enough, it depends on the neutron density instead of on the total number of neutrons.

The dispersion relation shows a displacement of the energy levels for the different modes, a negative shift for neutrinos, and a positive one for antineutrinos. The Dirac see level is displaced. Would the neutron star occupy the whole universe, it would just mean a change of variables, with no physical consequence. The finite size of the star changes the picture. Notice that b acts as the depth of a potential well [2]. It is repulsive for antineutrinos and attractive for neutrinos, which could condense (b plays also the role of a chemical potential).

The effective propagator corresponding to eq.(2) can be written as

$$\frac{i}{\hat{\not q}}, \tag{4}$$

with

$$\hat{\not q} = (q_0 - b)\,\gamma_0 \; - \; \vec{q}\,\vec{\gamma}, \tag{5}$$

an infrared safe propagator.

The above results could be obtained as well in an effective lagrangian approach. Since we are interested in long distance effects, and correspondingly low momentum exchange, it is a good approximation to use the following effective lagrangian, as done in ref. [1][3],

$$\mathcal{L}_{\text{eff}}(\vec{x}) = i\,\bar{\nu}\not\partial\,\nu(\vec{x}) - b\bar{\nu}\gamma_0\nu \ , \tag{6}$$

This effective lagrangian is valid inside the star. The second term summarizes the interactions with the neutrons, with the neutron current reduced in average to its charge density. For simplicity, we have assumed the neutron density to be constant in the star.

The effective lagrangien is not written in terms of quarks but in terms of a neutron density . Its validity is thus reduced to a momentum range larger than a typical hadronic size, $\sim 1\,\text{fm}$. When treating long distance effects, $\sim 1/R$, this implicit scale is irrelevant.

[2] When a neutrino-sea is present [4], the above remarks hold with b given by

$$b \simeq \mp\sqrt{2}G_F\,(n_n - n_\nu)/2 - \frac{8\sqrt{2}G_F\kappa}{3m_Z^2} n_\nu < E_\nu >,$$

where $< E_\nu >$ is the medium average of the neutrino energy, defined in the rest frame of the medium. These corrections to the fermion propagation would give higher order effects, and we disregard them in the present paper. Direct contributions of the neutrino sea to the energy level of the stars will be considered later, though.

[3] The connection with Fischbach's notation is given by the replacement $b = \sqrt{2}G_F a_n n_n$.

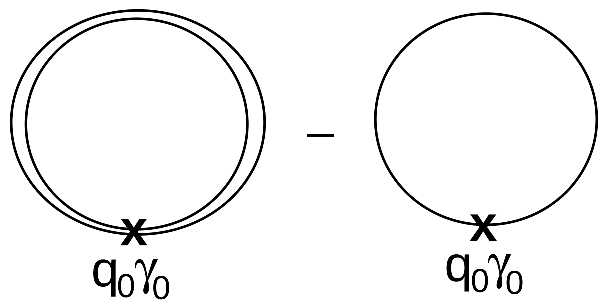

Figure 1: *Diagrammatic representation of eq. (8). The simple line represents the neutrino propagator in real vaccum. The double line stands for the "dressed" neutrino propagator, eq (4), which embodies the interactions with the medium.*

Eqs. (4) and (5) are the correct solution to the neutrino field equation inside the star which follows from eq. (6). Since the problematic potential energy under consideration corresponds to possible long distance divergences, it is worth to recall a well-known fact: eqs. (4) and (5) can be derived as well in perturbation by summing $1, 2 \ldots\ldots \infty$ interactions of an "undressed" neutrino with the neutrons. In the infrared, for $|\vec{q}| < b$, this perturbative method fails, though, because the infinite sum does not converge, it is infrared divergent. The derivation of eqs. (4) and (5) from eq. (6) has no such limitation. It has incorporated the interactions with the media, including the long distance ones. This suggests that no long distance divergence is to be expected from physical effects stemming from the interior of the star. We could stop the argument here. For the sake of comparison with previous literature, we explicit the computation below.

2 Energy density from neutrino exchange

A traditional way to estimate the energy induced by neutrino exchange is to compute first the possible $2, 3, 4, \ldots$ exchange potentials and then add their contributions integrated over the neutron positions in the star. We use instead a simpler and more direct method, formally equivalent to the latter, as proven in section 4.

As remarked by Fischbach, eq. (B2) in [1], Schwinger has provided the tools to compute the density of weak interaction energy, w, due to neutrino exchanges. It is given by the difference between the energy density for a neutrino propagating in the "vacuum" defined by the neutron star, $|\hat{0}>$, and the corresponding one for the real, matter-free, vacuum, $|0>$,

$$w = <\hat{0}|\mathcal{H}(0)|\hat{0}> - <0|\mathcal{H}_0(0)|0>, \tag{7}$$

where $\mathcal{H}_0(0)$ is the free hamiltonian density for the propagating neutrino, and $\mathcal{H}(0)$ includes the interaction with the neutrons, as given by eq. (6).

In diagramatic form, depicting the "dressed" massless fermion propagator, eq. (4), by a double line, the formal eq. (7) corresponds to the computation of the diagrams in fig. 1,

$$\int \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0\left(\frac{1}{\hat{\not{q}}}-\frac{1}{\not{q}}\right)L\right], \qquad L=\frac{1-\gamma_5}{2}. \tag{8}$$

This is a formal expression. A regularization procedure must be chosen. Let us consider Pauli-Villars. The problem has no infrared divergences, and it will control ultraviolet ones. Upon Pauli-Villars regularization, we can make a shift of variables, and the total result is zero, using the symmetry $q_\mu \to -q_\mu$.

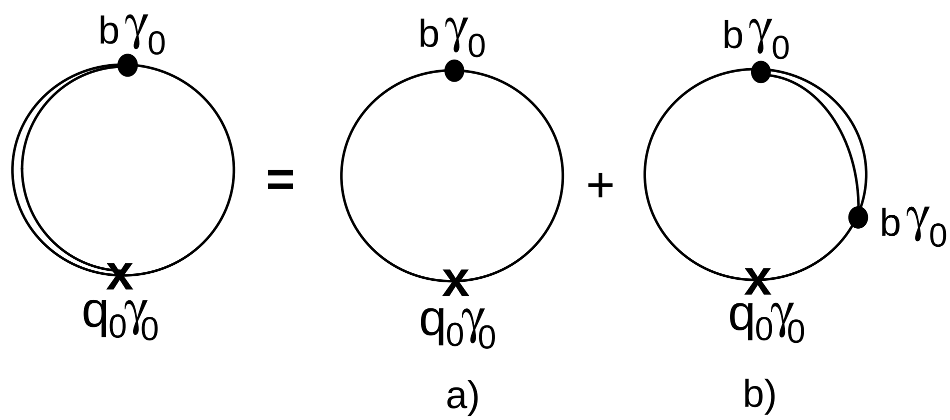

Figure 2: *The diagrams in fig. (1), developped according to Schwinger-Dyson. See eq. (10).*

We can gain further insight by using the Schwinger-Dyson expansion,

$$\frac{1}{\hat{\not{q}}}=\frac{1}{\not{q}}+\frac{1}{\not{q}}\;b\gamma_0\;\frac{1}{\hat{\not{q}}}, \tag{9}$$

and eq.(8) is equivalent to the computation of the diagrams in fig. 2,

$$w=\int \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0\,\frac{1}{\not{q}}b\gamma_0\,\frac{1}{\hat{\not{q}}}\,L\right]=\int \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0\,\frac{1}{\not{q}}b\gamma_0\,\frac{1}{\not{q}}\,L\right]+$$

$$\int \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0\,\frac{1}{\not{q}}b\gamma_0\,\frac{1}{\hat{\not{q}}}b\gamma_0\,\frac{1}{\not{q}},L\right]. \tag{10}$$

The first term on the right hand side of this equation corresponds in fact to the self-energy of one neutron, and strictly speaking it should be substracted (upon Pauli-Villars regularization, it gives a null contribution by itself, though). We see then that the neutrino contribution to the energy density stems formally from the computation of the diagram b) in fig.(2), that is,

$$w = \int_{PV} \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0 \frac{1}{\not q} b\gamma_0 \frac{1}{\hat{\not q}} b\gamma_0 \frac{1}{\not q} L\right] \;=\; 0, \tag{11}$$

where PV stands for Pauli-Villars. The result holds in any regularization scheme invariant for shifts of the energy variable.

3 Energy density from neutrino condensate

Before turning to the connection with Fischbach's work and the discussion of the regularization procedure, we should consider the presence of a neutrino sea inside the neutron star. This will be related to finite size effects. Indeed, Smirnov and Vissani have recently recalled that such a sea is present [4], and they argued that the two-body potential in the star is modified [5] due to the blocking effects of the sea, damping the long range forces.

We are interested in resumming the contribution to the energy from multibody exchange, to all orders. In our formalism this is easily done. It suffices to replace in eqs. (8)-(11) the dressed propagator, eq.(4), by

$$\widehat{S}_{\mathrm{F}} = i\left(\frac{1}{\hat{\not q}} + 2\pi i\, \hat{\not q}\, \delta(\widehat{q}^2)\theta(-q_0)\theta(q_0 - b)\right), \tag{12}$$

where the Fermi sea contains the neutrino states that have a negative energy in the star, i.e., according to eq. (2), $|\vec{q}| < |b|$ or $b < q_0 < 0$. As stated in [4] these neutrinos are trapped inside the star by the attractive potential while the antineutrinos are repelled away from the star.

Denoting by w_s this new contribution to the energy density, it follows that

$$w_s = \int \frac{d^4q}{(2\pi)^4}(-i)\mathrm{Tr}\left[q_0\gamma_0 \frac{1}{\not q} b\gamma_0\, 2\pi i \hat{\not q}\delta(\widehat{q}^{\,2})\theta(-q_0)\theta(q_0 - b)\, b\gamma_0 \frac{1}{\not q} L\right] =$$

$$+ \int \frac{d^3q}{(2\pi)^3}(b + |\vec{q}|)\theta(-b - |\vec{q}|) = -\frac{b^4}{24\pi^2} \sim -10^{-31}\,\mathrm{GeV\,fm}^{-3}. \tag{13}$$

This equation has an inmediate transparent interpretation: it is the contribution to the total energy density due to the neutrino Fermi-sea, which is supposed

to be filled up in a neutron star. Quantitatively, a tiny one. Its physical meaning stems from the finite size of the star, that is, from the difference of the energy levels between the dense medium and the free vacuum. In other words, would the neutron star occupy the whole universe, antineutrinos could not have been expelled out through its surface. Indeed, it is easy to check that w_s would vanish if a term representing an antineutrino sea would be added to the propagator, eq. (12).

Given the result in the previous section, the total energy density, eq. (7), is then given by

$$w = w_s \sim -10^{-31}\,\mathrm{GeV}\,\mathrm{fm}^{-3}. \tag{14}$$

4 Comparison with Fischbach's analysis

In order to compare with Fischbach analysis [1], eq.(11) can be developed using the Schwinger-Dyson equation (9). This leads to

$$w = \sum_{k=2}^{\infty} \int \frac{d^4q}{(2\pi)^4} (-i) \mathrm{Tr} \left[q_0 \gamma_0 \frac{1}{\not{q}} \left(b \gamma_0 \frac{1}{\not{q}} \right)^k L \right]. \tag{15}$$

Each term in the series of eq. (15) correspond exactly to those considered by Fischbach, with the difference that the sum does not stop at the total number of neutrons in the star, but runs up to ∞. It should be so: a given neutrino can interact several times with the same neutron, a fact not included in Fischbach treatment.

He has estimated the $k-$body potential for non-zero momentum transfer, and then integrated over the relative positions of the neutrons. This is equivalent to computing the loop in eq. (15) in which all of the k neutron current insertions have vanishing momentum transfer. With the method used in this paper, an insertion of the neutrino energy operator, $q_0\gamma_0$, is present as well. Upon integration by parts[4] with a proper regularization, it is straightforward to rewrite it as

$$w = \sum_{k=2}^{\infty} \frac{1}{k} \int \frac{d^4q}{(2\pi)^4} (-i) \mathrm{Tr} \left[\left(b \gamma_0 \frac{1}{\not{q}} \right)^k L \right]. \tag{16}$$

The factor $1/k$ appearing in this formula is the one required to obtain exactly the expansion of the logarithm, eq. (B34), in [1].

Taken separately, each term in the expansion (15) with 4 or more neutron insertions, $k \geq 4$, is infrared divergent, the degree of divergence rising with

[4]This integration by parts corresponds to the steps described in eqs. (B27)-(B29) in [1].

the number of neutron insertions. They would lead to a contribution to the energy density of the form $b^4(b\,R)^{k-4}$, where $b\,R \sim 1.10^{12} \gg 1$. This reasoning lead Fiscbach to conclude that neutron stars are unstable if the neutrinos are massless[5]. Nevertheless, as remarked above, the correct behaviour which follows from the field equations for the lagrangian, eq.(6), proves that the sum of the series is infrared convergent, as evident from eq. (11). We are facing a situation where the behaviour of the different terms in a perturbative expansion differs essentially from the resummed, non-perturbative, result[6]. *Multibody massless neutrino exchange do not destabilize the star.*

For the sake of comparison with Fischbach's paper it is worth to comment upon the treatment of ultraviolet divergences, even if the latter are irrelevant to the long distance problem under discussion, as already remarked in [1]. As finally such a problem does not exist, they could be the leading contribution to the star self-energy, although a harmless one.

Fischbach has correctly remarked that only terms with 4 or less neutron insertions, $k \leq 4$, are ultraviolet divergent. He explicitly computed their finite contribution to the star self-energy. For $k = 2$ he obtained a non-zero result, contrary to our results above. The issue is not the form of the 2-body exchange potential, but the energy density derived from it upon spatial integration, where the ultraviolet divergence appears. The difference stems from the regulator choice. For simplicity, we have chosen above to work with Pauli-Villars, a perfectly consistent one. Fischbach implemented a cutoff non invariant for the shift of energy variables, a harder one, leading to a finite result. He points out that there is a natural ultraviolet cutoff in the problem - the "hard core" r_c,

$$r_c \simeq 0.5 \text{ fm}, \tag{17}$$

which prevents neutrons from "piling up" in space. It can be interpreted as the natural hadronic cutoff for the effective lagrangien, eq. (6), discussed above. To consider the effects of such a cutoff is tantamount to go beyond the Hartree Fock aproximation, where correlations among neutrons are neglected. r_c is then to be added to our list of physically relevant parameters in eq. (1). Its exact shape is not known from first principles. We do not wish to discuss here Fischbach's choice, as it only affects the multibody potentials involving 4 neutrons or less, irrelevant for the infrared problem under discussion. The physical consequences, if present, should be derivable from the underlying theory, the standard model. We do not follow this line of research here, as the main focus of this paper are

[5]Smirnov and Vissani, [5] revised version may 24th, argue that due to Pauli blocking, the infrared dangerous parameter $O(bR)$ in Fischbach's expansion is replaced by a parameter $O(1)$ and the series might be resummed. We have performed the sum with (section 3), or without (section 2) Pauli blocking and find no infrared divergence.

[6]From an algebraic point of view the behavior is similar to that of the function $1/(1-x)$, for $x > 1$, which is wellfined even if its expansion in x diverges.

long distance effects, for which r_c is irrelevant. His result, even if larger than the contribution of the Fermi sea derived here, does not threaten the stability of the star. Would we use Fischbach's cutoff, our dominant contribution would be provided by the term with the strongest ultraviolet divergence, $k = 2$: from two body neutrino exchange. The result is, eq. (4.2) in[7] [1],

$$w^{(2)} = \frac{b^2}{8\pi^2 r_c^2} \sim 10^{-16}\,\mathrm{GeV\,fm^{-3}}, \tag{18}$$

to be compared to the neutron mass density $\sim O(1\ \mathrm{GeV\ fm^{-3}})$. Eq. (18) does not take into account the presence of the neutrino sea. As the latter only corrects the long distance behaviour, the minor changes resulting from its inclusion do not change the physical result for the star energy density.

5 Conclusion

We have shown that the resummation of multibody neutrino exchange in a neutron star results in an infrared well-behaved contribution to the star energy density. This holds whether the presence of a neutrino sea is taken into account or not. The resulting energy density is negligible as compared to the star mass density. The star remains stable, even if neutrinos are massless.

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